

$$(x + 1)^3 = x^3 + 3x^2 + 3x + 1$$

$$(x + 1)^3 - x^3 = 3x^2 + 3x + 1$$

$$\sum_{i=1}^n (x + 1)^3 - x^3 = \sum_{i=1}^n 3x^2 + 3x + 1$$

$$(n + 1)^3 - 1 = 3 \sum_{i=1}^n x^2 + 3 \sum_{i=1}^n x + \sum_{i=1}^n 1$$

$$\sum_{i=1}^n x^2 = \frac{(n + 1)^3 - \sum 3x - \sum 1 - 1}{3}$$

$$\sum_{i=1}^n x^2 = \frac{(n^3 + 3n^2 + 3n + 1) - 3 \sum x - \sum 1 - 1}{3}$$

$$\sum_{i=1}^n x^2 = \frac{(n^3 + 3n^2 + 3n + 1) - \frac{3n(n + 1)}{2} - n - 1}{3}$$

$$\sum_{i=1}^n x^2 = \frac{2(n^3 + 3n^2 + 3n + 1) - 3n(n + 1) - 2n - 2}{6}$$

$$\sum_{i=1}^n x^2 = \frac{2n^3 + 3n^2 + n}{6}$$

$$\sum_{i=1}^n x^2 = \frac{n(n + 1)(2n + 1)}{6}$$