VECTOR DERIVATIVES

CARTESIAN: $d\mathbf{l} = dx\,\hat{\mathbf{x}} + dy\,\hat{\mathbf{y}} + dz\,\hat{\mathbf{z}}$; $\quad d\mathbf{r} = dx\,dy\,dz$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial x}\hat{\mathbf{x}} + \dfrac{\partial T}{\partial y}\hat{\mathbf{y}} + \dfrac{\partial T}{\partial z}\hat{\mathbf{z}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{\partial v_x}{\partial x} + \dfrac{\partial v_y}{\partial y} + \dfrac{\partial v_z}{\partial z}$

Curl $\quad \nabla \times \mathbf{v} = \hat{\mathbf{x}}\left(\dfrac{\partial v_z}{\partial y} - \dfrac{\partial v_y}{\partial z}\right) + \hat{\mathbf{y}}\left(\dfrac{\partial v_x}{\partial z} - \dfrac{\partial v_z}{\partial x}\right) + \hat{\mathbf{z}}\left(\dfrac{\partial v_y}{\partial x} - \dfrac{\partial v_x}{\partial y}\right)$

Laplacian $\quad \nabla^2 T = \dfrac{\partial^2 T}{\partial x^2} + \dfrac{\partial^2 T}{\partial y^2} + \dfrac{\partial^2 T}{\partial z^2}$

SPHERICAL: $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r d\theta\,\hat{\boldsymbol{\theta}} + r\sin\theta\,d\phi\,\hat{\boldsymbol{\phi}}$; $\quad d\mathbf{r} = dl_r dl_\theta dl_\phi = r^2\sin\theta\,dr d\theta d\phi$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial T}{\partial \theta}\hat{\boldsymbol{\theta}} + \dfrac{1}{r\sin\theta}\dfrac{\partial T}{\partial \phi}\hat{\boldsymbol{\phi}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2 v_r\right) + \dfrac{1}{r\sin\theta}\dfrac{\partial}{\partial \theta}\left(\sin\theta\,v_\theta\right) + \dfrac{1}{r\sin\theta}\dfrac{\partial v_\phi}{\partial \phi}$

Curl

$$\nabla \times \mathbf{v} = \frac{1}{r\sin\theta}\left[\frac{\partial}{\partial\theta}(\sin\theta v_\phi) - \frac{\partial v_\theta}{\partial\phi}\right]\hat{\mathbf{r}} + \frac{1}{r}\left[\frac{1}{\sin\theta}\frac{\partial v_r}{\partial\phi} - \frac{\partial}{\partial r}(r v_\phi)\right]\hat{\boldsymbol{\theta}} + \frac{1}{r}\left[\frac{\partial}{\partial r}(r v_\theta) - \frac{\partial v_r}{\partial\theta}\right]\hat{\boldsymbol{\phi}}$$

Laplacian $\quad \nabla^2 T = \dfrac{1}{r^2}\dfrac{\partial}{\partial r}\left(r^2\dfrac{\partial T}{\partial r}\right) + \dfrac{1}{r^2\sin\theta}\dfrac{\partial}{\partial\theta}\left(\sin\theta\dfrac{\partial T}{\partial\theta}\right) + \dfrac{1}{r^2\sin^2\theta}\dfrac{\partial^2 T}{\partial\phi^2}$

CYLINDRICAL: $d\mathbf{l} = dr\,\hat{\mathbf{r}} + r d\phi\,\hat{\boldsymbol{\phi}} + dz\,\hat{\mathbf{z}}$; $\quad d\mathbf{r} = r dr\,d\phi\,dz$

Gradient $\quad \nabla T = \dfrac{\partial T}{\partial r}\hat{\mathbf{r}} + \dfrac{1}{r}\dfrac{\partial T}{\partial\phi}\hat{\boldsymbol{\phi}} + \dfrac{\partial T}{\partial z}\hat{\mathbf{z}}$

Divergence $\quad \nabla \cdot \mathbf{v} = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r v_r\right) + \dfrac{1}{r}\dfrac{\partial v_\phi}{\partial\phi} + \dfrac{\partial v_z}{\partial z}$

Curl $\quad \nabla \times \mathbf{v} = \left(\dfrac{1}{r}\dfrac{\partial v_z}{\partial\phi} - \dfrac{\partial v_\phi}{\partial z}\right)\hat{\mathbf{r}} + \left(\dfrac{\partial v_r}{\partial z} - \dfrac{\partial v_z}{\partial r}\right)\hat{\boldsymbol{\phi}} + \dfrac{1}{r}\left(\dfrac{\partial}{\partial r}(r v_\phi) - \dfrac{\partial v_r}{\partial\phi}\right)\hat{\mathbf{z}}$

Laplacian $\quad \nabla^2 T = \dfrac{1}{r}\dfrac{\partial}{\partial r}\left(r\dfrac{\partial T}{\partial r}\right) + \dfrac{1}{r^2}\dfrac{\partial^2 T}{\partial\phi^2} + \dfrac{\partial^2 T}{\partial z^2}$

VECTOR IDENTITIES

TRIPLE PRODUCTS

(1) $\mathbf{A}\cdot(\mathbf{B}\times\mathbf{C}) = \mathbf{B}\cdot(\mathbf{C}\times\mathbf{A}) = \mathbf{C}\cdot(\mathbf{A}\times\mathbf{B})$

(2) $\mathbf{A}\times(\mathbf{B}\times\mathbf{C}) = \mathbf{B}(\mathbf{A}\cdot\mathbf{C}) - \mathbf{C}(\mathbf{A}\cdot\mathbf{B})$

PRODUCT RULES

(3) $\nabla(fg) = f\nabla g + g\nabla f$

(4) $\nabla(\mathbf{A}\cdot\mathbf{B}) = \mathbf{A}\times(\nabla\times\mathbf{B}) + \mathbf{B}\times(\nabla\times\mathbf{A}) + (\mathbf{A}\cdot\nabla)\mathbf{B} + (\mathbf{B}\cdot\nabla)\mathbf{A}$

(5) $\nabla\cdot(f\mathbf{A}) = f(\nabla\cdot\mathbf{A}) + \mathbf{A}\cdot(\nabla f)$

(6) $\nabla\cdot(\mathbf{A}\times\mathbf{B}) = \mathbf{B}\cdot(\nabla\times\mathbf{A}) - \mathbf{A}\cdot(\nabla\times\mathbf{B})$

(7) $\nabla\times(f\mathbf{A}) = f(\nabla\times\mathbf{A}) - \mathbf{A}\times(\nabla f)$

(8) $\nabla\times(\mathbf{A}\times\mathbf{B}) = (\mathbf{B}\cdot\nabla)\mathbf{A} - (\mathbf{A}\cdot\nabla)\mathbf{B} + \mathbf{A}(\nabla\cdot\mathbf{B}) - \mathbf{B}(\nabla\cdot\mathbf{A})$

SECOND DERIVATIVES

(9) $\nabla\cdot(\nabla\times\mathbf{v}) = 0$

(10) $\nabla\times(\nabla T) = 0$

(11) $\nabla\times(\nabla\times\mathbf{v}) = \nabla(\nabla\cdot\mathbf{v}) - \nabla^2\mathbf{v}$

FUNDAMENTAL THEOREMS

Gradient Theorem $\int_{\mathbf{a}}^{\mathbf{b}} (\nabla T)\cdot d\mathbf{l} = T(\mathbf{b}) - T(\mathbf{a})$

Divergence Theorem $\int_{volume} (\nabla\cdot\mathbf{v})d\mathbf{r} = \oint_{surface} \mathbf{v}\cdot d\mathbf{a}$

Curl Theorem $\int_{surface} (\nabla\times\mathbf{v})\cdot d\mathbf{a} = \oint_{boundary\ line} \mathbf{v}\cdot d\mathbf{l}$

Coulomb's Law $\mathbf{F} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{qQ}{r^2}\hat{\mathbf{r}}$

Electric Field $\mathbf{F} = Q\mathbf{E}$ $\mathbf{E} = \dfrac{1}{4\pi\varepsilon_0}\int \dfrac{\rho(r)}{r^2}\hat{\mathbf{r}}d\mathbf{r}$

Gauss' Law $\int \mathbf{E}\cdot d\mathbf{a} = \dfrac{Q_{enclosed}}{\varepsilon_0}$ $\nabla\cdot\mathbf{E} = \dfrac{\rho}{\varepsilon_0}$

Electric Potential $V(\mathbf{b}) - V(\mathbf{a}) = -\int_{\mathbf{a}}^{\mathbf{b}} \mathbf{E} \cdot d\mathbf{l}$ $\mathbf{E}(\mathbf{r}) = -\nabla V(\mathbf{r})$

Poisson' Eqn $\nabla^2 V = -\frac{\rho}{\varepsilon_0}$ $V = \frac{1}{4\pi\varepsilon_0} \int \frac{\rho(r)}{r} d\mathbf{r}$

Laplace's Eqn $\nabla^2 V = 0$ $V(r,\theta) = \sum_{l=0}^{\infty} \left(A_l r^l + \frac{B_l}{r^{l+1}} \right) P_l(\cos\theta)$

Boundary Conditions $V_{above} = V_{below}$ $\frac{\partial V_{above}}{\partial n} - \frac{\partial V_{below}}{\partial n} = -\frac{\sigma}{\varepsilon_0}$

Legendre polynomials $P_0(x) = 1$ $P_1(x) = x$ $P_2(x) = (3x^2 - 1)/2$

$P_3(x) = (5x^3 - 3x)/2$ $P_4(x) = (35x^4 - 30x^2 + 3)/8$

Stored energy $W = \frac{1}{2} \sum_{i=1}^{n} q_i V(\mathbf{r}_i)$ $W = \frac{1}{2} \int \rho(\mathbf{r}) V(\mathbf{r}) d\mathbf{r} = \frac{\varepsilon_0}{2} \int E^2 d\mathbf{r}$

Bound charge densities $\rho_b = -\nabla \cdot \mathbf{P}$ $\sigma_b = \mathbf{P} \cdot \hat{\mathbf{n}}$

Electric Displacement $\mathbf{D} = \varepsilon_0 \mathbf{E} + \mathbf{P}$ $\oint \mathbf{D} \cdot d\mathbf{a} = Q_{f\,\text{enclosed}}$

Linear Dielectrics $\mathbf{P} = \chi_e \varepsilon_0 \mathbf{E}$ $\mathbf{D} = \varepsilon_0 (1 + \chi_e) \mathbf{E} = \varepsilon \mathbf{E}$

$\varepsilon = \varepsilon_r \varepsilon_0 = (1 + \chi_e) \varepsilon_0$

Boundary Conditions $\mathbf{D}^{\parallel}_{above} - \mathbf{D}^{\parallel}_{below} = \mathbf{P}^{\parallel}_{above} - \mathbf{P}^{\parallel}_{below}$ $D^{\perp}_{above} - D^{\perp}_{below} = \sigma_f$

Capacitance $C = \frac{Q}{V}$

Biot-Savart $\mathbf{B}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}' \times \hat{\mathbf{s}}}{s^2}$

Ampere's Law $\oint \mathbf{B} \cdot d\mathbf{l} = \mu_0 I_{enc}$ $\nabla \times \mathbf{B} = \mu_0 \mathbf{J}$

Vector Potential $\mathbf{B} = \nabla \times \mathbf{A}$ $\mathbf{A}(\mathbf{r}) = \frac{\mu_0 I}{4\pi} \int \frac{d\mathbf{l}'}{s}$

Magnetic Forces $\mathbf{F} = q\mathbf{v} \times \mathbf{B}$ $d\mathbf{F} = I d\mathbf{l} \times \mathbf{B}$

Bound current densities $\mathbf{J}_b = \nabla \times \mathbf{M}$ $\mathbf{K}_b = \mathbf{M} \times \hat{\mathbf{n}}$

Boundary Conditions $\mathbf{B}_{above} - \mathbf{B}_{below} = \mu_0 (\mathbf{K} \times \hat{\mathbf{n}})$

$\mathbf{A}_{above} = \mathbf{A}_{below}$ $\frac{\partial}{\partial n} \mathbf{A}_{above} - \frac{\partial}{\partial n} \mathbf{A}_{below} = -\mu_0 \mathbf{K}$

Linear Media $\mathbf{M} = \chi_m \mathbf{H}$ $\mathbf{B} = \mu_0(1+\chi_m)\mathbf{H} = \mu\mathbf{H}$

$\mu = \mu_r \mu_0 = (1+\chi_m)\mu_0$

Stored Energy $W = \dfrac{1}{2\mu}\int B^2 d\mathbf{r}$

Faraday's Law $\oint \mathbf{E}\cdot d\mathbf{l} = -\dfrac{d\phi}{dt}$ $\phi = \int_S \mathbf{B}\cdot d\mathbf{a}$

Inductance $M_{21} = \dfrac{\mu_0}{4\pi}\oint\oint \dfrac{d\mathbf{l}_1 \cdot d\mathbf{l}_2}{s}$ $\varepsilon = -L\dfrac{dI}{dt}$

Maxwell's Equations

$$\nabla\cdot\mathbf{E} = \frac{\rho}{\varepsilon_0} \qquad \nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t} \qquad \nabla\times\mathbf{B} = \mu_0\left(\mathbf{J} + \varepsilon_0\frac{\partial\mathbf{E}}{\partial t}\right)$$

Polarization Current $\mathbf{J}_p = \dfrac{\partial\mathbf{P}}{\partial t}$

Maxwell's Equations in Matter

$$\nabla\cdot\mathbf{D} = \rho_f$$

$$\nabla\cdot\mathbf{B} = 0$$

$$\nabla\times\mathbf{E} = -\frac{\partial\mathbf{B}}{\partial t}$$

$$\nabla\times\mathbf{H} = \mathbf{J}_f + \frac{\partial\mathbf{D}}{\partial t}$$

for linear media $\left.\begin{matrix}\mathbf{P} = \varepsilon_0\chi_e\mathbf{E} \\ \mathbf{M} = \chi_m\mathbf{H}\end{matrix}\right\}$ so that $\left.\begin{matrix}\mathbf{D} = \varepsilon\mathbf{E} \\ \mathbf{H} = \frac{1}{\mu}\mathbf{B}\end{matrix}\right\}$