Graph Theory and Applications
Dr. Ayşegül Yayımlı

Beycan Kahraman
504071508

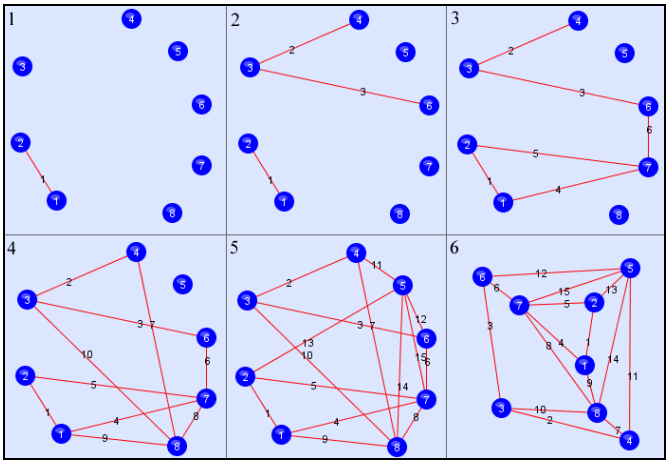
Homework 2 **07.11.2008**

1. Which of the following partitions are graphical? Draw the graphs if possible.
a. 8, 7, 6, 5, 4, 3, 2, 2, 1
b. 5, 5, 5, 3, 3, 3, 3, 3

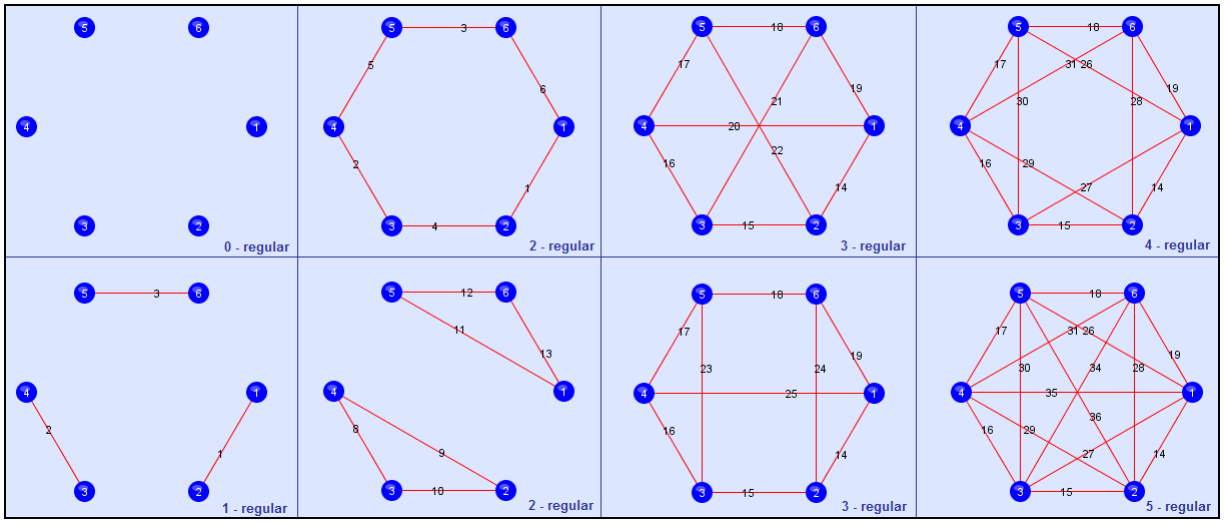
a. 876543221
65432110
4321000 Therefore, not graphical.

b) 55533333
4422233 -> 4433222
322122 -> 322221
11121 -> 21111
0011 -> 1100
00 Therefore, graphical.

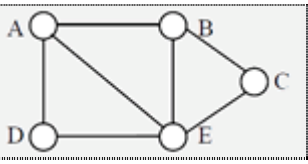
The graph could be drawn as shown at the figure.



2. Construct all regular graphs with six vertices.



3.
a. Find the number of spanning trees in the following graph:
b. Draw those spanning trees.



a.

	A	B	C	D	E
A	3	0	0	0	0
B	0	3	0	0	0
C	0	0	2	0	0
D	0	0	0	2	0
E	0	0	0	0	4

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	A	B	C	D	E
A	0	1	0	1	1
B	1	0	1	0	1
C	0	1	0	0	1
D	1	0	0	0	1
E	1	1	1	1	0

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	A	B	C	D	E
A	3	-1	0	-1	-1
B	-1	3	-1	0	-1
C	0	-1	2	0	-1
D	-1	0	0	2	-1
E	-1	-1	-1	-1	4

M =

	A	B	C	D
A	3	-1	0	-1
B	-1	3	-1	0
C	0	-1	2	0
D	-1	0	0	2

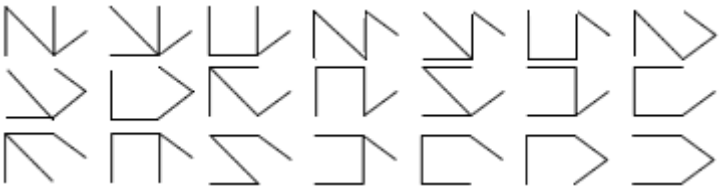
Then,

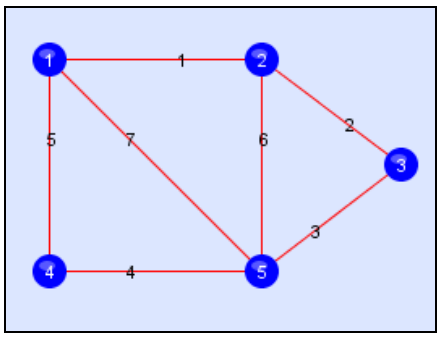
det (M) = 3(12-2)+(-4-2)+(-6+1) = 21

Therefore, there are 21 different spanning trees.

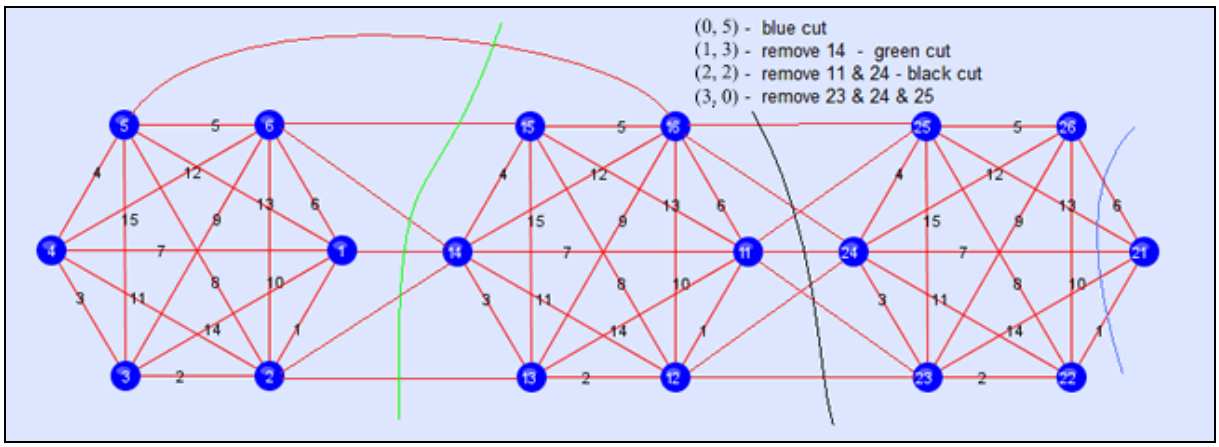
b. Edges to be deleted in order to form 21 different spanning trees:

1,2,4 - 1,2,5 - 1,2,7 - 1,3,4 - 1,3,5 - 1,3,7 - 1,4,6
1,5,6 - 1,6,7 - 2,4,6 - 2,4,7 - 2,5,6 - 2,5,7 - 2,6,7
3,4,6 - 3,4,7 - 3,5,6 - 3,5,7 - 3,6,7 - 4,6,7 - 5,6,7





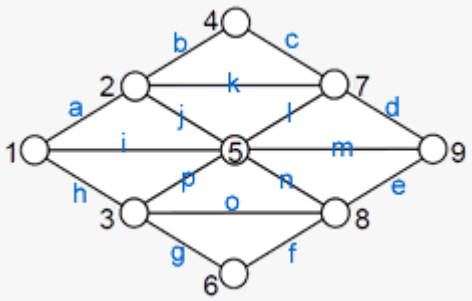
4. Find a graph with vertices s, and t, where the connectivity function for these vertices is:
(0,5), (1,3), (2,2), (3,0)



5. Consider the graph shown on the right.

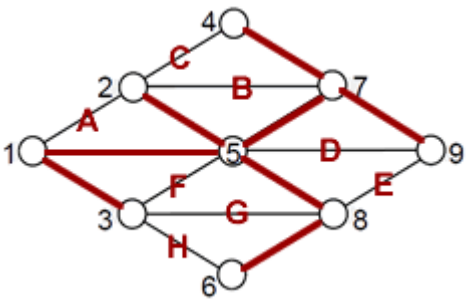
a. List all fundamental cycles with respect to the following spanning tree:
T = {(1,3), (1,5), (2,5), (4,7), (5,7), (5,8), (6,8), (7,9)}
Which fundamental cycles form the following cycles of the graph, by ringsum operation?
K(1,2,4,7,9,8,6,3) L(2,7,5,8,3,1)

b. For the spanning tree given, list all fundamental cuts. Which of these are (1,9)-cuts?



a. All 8 fundamental cycles are (nodes in the cycle):
A(1, 5, 2), B(2, 5, 7), C(2, 5, 7, 4), D(5, 7, 9), E(8, 5, 7, 9),
F(3, 1, 5), G(3, 1, 5, 8), H(3, 1, 5, 8, 6).

K = A ⊕ C ⊕ E ⊕ H L = A ⊕ B ⊕ G



b. All 8 fundamental cuts are:
[1-3](h, p, o, g), [1-5](a, i, p, o, g), [2-5](b, k, j, a), [4-7](b, c), [5-7](b, k, l, m, e), [5-8](g, o, n, e), [6-8](g, f), [7-9](d, m, e).

All 3 of the (1,9)-cuts are [1-5], [5-7], [7-9].