Mathematical Analysis of IEEE 802.11e EDCA

p_i: Attempt probability of class i

$$p_i = \frac{2(1-2c_i)}{(CW_{min,i}-1)(1-2c_i)+CW_{min,i}c_i(1-(2c_i)^{m_i})}$$

c_i: Collision probability of class i

$$c_i = 1-(1-p_i)^{N_i-1}\prod_{j\neq i}(1-p_j)^{N_j}$$

π_i: stationary probability of being in zone i

$$c_i = \frac{\pi_i}{\sum_{j=i}^{K}\pi_j}(1-(1-p_1)^{N_1}\ldots(1-p_i)^{N_i-1})+\cdots+\frac{\pi_K}{\sum_{j=i}^{K}\pi_j}(1-(1-p_i)^{N_i-1}\ldots(1-p_K)^{N_K})$$

γ_i: Throughput of class i

$$\gamma_i = \frac{\pi_A\sigma_A p_A(1-p_A)^{N_A-1}+\pi_B\sigma_B p_B(1-p_B)^{N_B-1}(1-p_A)^{N_A}}{\pi_A E[S_A]+\pi_B E[S_B]}$$

γ_i: Collision probability of class i, β_i: Attempt probability of class i

$$\beta_i = G_i(\gamma_i) = \frac{1+\gamma_i+\gamma_i^2+\cdots+\gamma_i^{K_i}}{b_{i,0}+b_{i,1}\gamma_i+b_{i,2}\gamma_i^2+\cdots+b_{i,K}\gamma_i^{K_i}}$$

$$\gamma_i = \Gamma(\beta_1,\beta_2,\ldots,\beta_n) = 1-\prod_{j=1,j\neq i}^{n}(1-\beta_j)$$

q_{EA}: The probability that the system is in the Excess AIFS mode (only high priority user)

$$q_{EA} = \prod_{i=1}^{n^{(1)}}\left(1-\beta_i^{(1)}\right)$$

q_R: The probability that the system is in the remaining AIFS zones

$$q_R = \prod_{i=1}^{n^{(1)}}\left(1-\beta_i^{(1)}\right)\prod_{i=j}^{n^{(0)}}\left(1-\beta_j^{(0)}\right)$$

π_i: Stationary probability of being in zone i

$$\pi_{EA} = \frac{1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1}}{1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1}+q_{EA}^{l}/(1-q_R)}$$

$$\pi_R = \frac{q_{EA}^{l}/(1-q_R)}{1+q_{EA}+q_{EA}^2+\cdots+q_{EA}^{l-1}+q_{EA}^{l}/(1-q_R)}$$

Thus, the collision probabilities of the classes:

$$\gamma_i^{(1)} = \pi_{EA}\left(1-\prod_{j=1,j\neq i}^{n^{(1)}}\left(1-\beta_j^{(1)}\right)\right)+\pi_R\left(1-\left(\prod_{j=1,j\neq i}^{n^{(1)}}\left(1-\beta_j^{(1)}\right)\prod_{j=1}^{n^{(0)}}\left(1-\beta_j^{(0)}\right)\right)\right)$$

$$\gamma_i^{(0)} = 1-\left(\prod_{j=1}^{n^{(1)}}\left(1-\beta_j^{(1)}\right)\prod_{j=1,j\neq i}^{n^{(0)}}\left(1-\beta_j^{(0)}\right)\right)$$

τ: Transmit probability

$$\tau = \frac{2(1-2p)}{(1-2p)(W+1) + pW(1-(2p)^m)}$$

p: Conditional collision probability

$$p = 1 - (1-\tau)^{n-1}$$

P_{tr} : Probability of transmission on a channel

$$P_{tr} = 1 - (1-\tau)^n$$

P_s: Successful transmission probability

$$P_s = \frac{n\tau(1-\tau)^{n-1}}{P_{tr}} = \frac{n\tau(1-\tau)^{n-1}}{1-(1-\tau)^n}$$

S: Normalized system throughput

$$S = \frac{E[payload\ information]}{E[length\ of\ slot\ time]} = \frac{P_s P_{tr}\, E[P]}{(1-P_{tr})\sigma + P_s P_{tr} T_s + P_s(1-P_{tr})T_c}$$

σ: slot time, T_s: duration of successful transmission, T_c: duration of collision, $E[P]$: expected packet length

γ: attempts, β: collision probability

$$\beta = G(\gamma) = \frac{E[R]}{E[X]} = \frac{1 + \gamma + \gamma^2 + \cdots + \gamma^K}{b_0 + b_1\gamma + b_2\gamma^2 + \cdots + b_K\gamma^K}$$

$E[R]$: expected attempts, $E[X]$: expected renewal cycle, b_i: CW size of ith step

$$\gamma = \Gamma(\beta) = 1 - (1-\beta)^{n-1}$$

For unbalanced fixed point case:

$$\gamma_i = 1 - \prod_{j=1, j\neq i}^{n} \left(1 - G(\gamma_j)\right)$$

π_i: Stationary probability of being in zone i

$$\pi_A = \frac{1 + q_A + q_A^2 + \cdots + q_A^{l-1}}{1 + q_A + q_A^2 + \cdots + q_A^{l-1} + q_A^l/(1-q_B)}$$

$$\pi_B = \frac{q_A^l/(1-q_B)}{1 + q_A + q_A^2 + \cdots + q_A^{l-1} + q_A^l/(1-q_B)}$$

q_i: The probability that the slot is idle in zone i

$P_{i,zoneJ}$: The probability of being in zone J for class i

$$P_{A,zone\ 1} = 1 - (1-\tau_A)^{n_A}$$

$$P_{A,zone\ 2} = 1 - (1-\tau_A)^{n_A-1}(1-\tau_B)^{n_B}$$

$$P_{B,zone\ 2} = 1 - (1-\tau_A)^{n_A}(1-\tau_B)^{n_B-1}$$

P^{tr}_{zoneJ} : The probability of at least one transmission in zone J

$$P^{tr}_{zone\ 1} = 1 - (1-\tau_A)^{n_A}$$

$$P^{tr}_{zone\ 2} = 1 - (1-\tau_A)^{n_A}(1-\tau_B)^{n_B}$$

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